LA-8204-MS

Informal Report

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Developmental Studies of Constitutive Models for Plastic-Bonded Explosives

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UNITED STATES
DEPARTMENT OF ENERGY
CONTRACT W-7405-ENG. 36

LA-8204-MS
Informal Report
UC-45
Issued: June 1980

Developmental Studies of Constitutive Models for Plastic-Bonded Explosives

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CONTENTS

ABSTRACT 1

I. INTRODUCTION 1

II. MODELING TECHNIQUES 2
A. Creep Model Formation 2
B. Viscoelastic Model Formation 5

III. SUMMARY AND CONCLUSIONS 8

ACKNOWLEDGMENT 8

REFERENCES 8

DEVELOPMENTAL STUDIES OF CONSTITUTIVE MODELS FOR PLASTIC-BONDED EXPLOSIVES

by

Robert M. Hackett, Randall L. Peeters, and William R. Oakes, Jr.

ABSTRACT

Typically, elastic and elastic-plastic theory are used in structural analysis computer programs to model the mechanical behavior of high explosives; these models, however, do not fit the observed behavior of plastic-bonded explosives. This report discusses the development of an equation-of-state creep model and a linear viscoelastic model for the analysis of these material systems and shows comparisons between experimental results and analytical model predictions.

I. INTRODUCTION

We require accurate constitutive models of the individual weapon components for the development of modern weapon systems. These models are necessarily complicated because of irregular geometries and the nonlinear nature of the different materials. Highly developed stress analysis computer codes capable of modeling three-dimensional geometries and loading conditions are available and are being used at the Los Alamos Scientific Laboratory (LASL). However, the results obtained with the codes cannot be relied upon unless the constitutive behavior of the component materials is well understood and modeled.

In predicting the response of systems utilizing plastic-bonded high explosives (HE), the characterization of the HE material is of paramount importance. To date, little information concerning the mechanical response of this class of materials is available. Experimental programs have shown the material to be highly complex and somewhat unique in comparison to other materials that exhibit the same general physical characteristics. For example, the high-solids loading with a plastic binder results in a stiff polymeric material characterized by small strain capability, combined with a creep-like response. Therefore, the task of developing suitable constitutive models to describe the response of the HE materials, and to subsequently be used in computer codes to predict overall mechanical-thermal system response, is a formidable one. This report describes some of the initial efforts at LASL to define the critical modeling parameters. These models are mathematically sound and yield results that correlate with experimental observatons. The analytical developments cited in the report are based primarily on material testing already conducted at LASL. The described analytical work can also help define the direction and role of further testing. A continued analytical/experimental interactive development program of this nature is clearly warranted, based upon the results of the initial work reported herein.

II. MODELING TECHNIQUES

In the development described, the initial modeling is based upon a postulated elastic-plastic thermal creep response. The models to be described do not portray this complete phenomenon, but are developed in such a manner to be contained in this domain. For example, although the models presented are formulated without a consideration of thermal effect, such a consideration can be incorporated subsequently without reconsideration of the complete basis of the formulation. On the other hand, a consideraton of "damage," which goes beyond the conception of a "deformable" continuum, would, in all likelihood, require a significant reformulation of the considered models. In fact, as will be seen, reasons to consider a theroy that includes damage in the model formulation may exist.

The present analytical formulation is based upon a characterization of the time-dependent material behavior. Three accepted methods of modeling this behavior are: (1) the phenomenological or equation-of-state creep theory, (2) the memory or hereditary theory, and (3) the nonlinear viscoelastic aproach. The analytical studies described here are based upon methods 1 and 3. All of the data used in the formulations are based upon room temperature testing of an inert plastic bonded material, designated 900-10.

A. Creep Model Formulation

Initial studies centered on the equation-of-state creep theory. Creep models of this type are used in nuclear reactor technology[1] to predict the deformation, under sustained loading, of metal structures subjected to high temperatures. Such models exist in numerous nonlinear finite element computer codes, including ADINA[2] (Automatic Dynamic Incremental Nonlinear Analysis), which is presently employed at LASL. The two one-dimensional creep models in ADINA have the following mathematical forms.

$$\epsilon_c = a_0 \sigma^{a_1} t^{a_2}, \qquad (1)$$

where ϵ_c is the uniaxial creep strain, σ is the uniaxial stress at a sustained load, t is the time since the application of the sustained load, and a_0 through a_2 are material parameters obtained from a series of uniaxial creep tests conducted at different stress levels; and

$$\epsilon_c = F\,[1-\exp(-Rt)] + Gt, \qquad (2)$$

where

$$F = a_0 \exp(a_1 \sigma), \qquad (3a)$$

$$R = a_2 \left(\frac{\sigma}{a_3}\right)^{a_4}, \qquad (3b)$$

and

$$G = a_5 \exp(a_6 \sigma), \qquad (3c)$$

where a_0 through a_6 are material parameters. The ADINA code also provides for the inclusion of a "user-developed" third creep model.

By using creep data obtained from a series of four creep tests, shown in Figs. 1 and 2, an attempt was made to model the response of the inert material using Eq. (1) and Eq. (2).* Early results indicated that Eq. (1) could not be employed to model the inert material. The parameter a_2 from Eq. (1), as determined from the recorded creep data, has a value less than unity that is not admissable in the creep strain rate calculation step in ADINA. It was also found, from early investigation, that Eq. (2), when provided with values of the constants a_0 through a_6 from the creep test data, could not reproduce load-deflection histories. Thus, it was concluded that Eq. (1) and Eq. (2) were inadequate as creep models for the characterization of the inert material.

Two new creep models were then developed for the ADINA code. Both of these models were based upon the creep test results displayed in Figs. 1 and 2. They were coded into the Hewlett-Packard finite element package, and an attempt was made to analytically predict the three constant-strain rate-to-failure curves for the inert material shown in Fig. 4.

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*In order to quickly perform the necessary parameter studies, an efficient single-element finite element code was written for the Hewlett-Packard 9825A calculator. The program utilizes the four-node isoparametric axisymmetric quadrilateral element and the same elastic-plastic thermal-creep analysis found in the ADINA code. Experimental creep data is read from a flexible disc; the experimental and predicted response curves are plotted on a multicolor x-y recorder. Figure 3 shows the various components in the system.

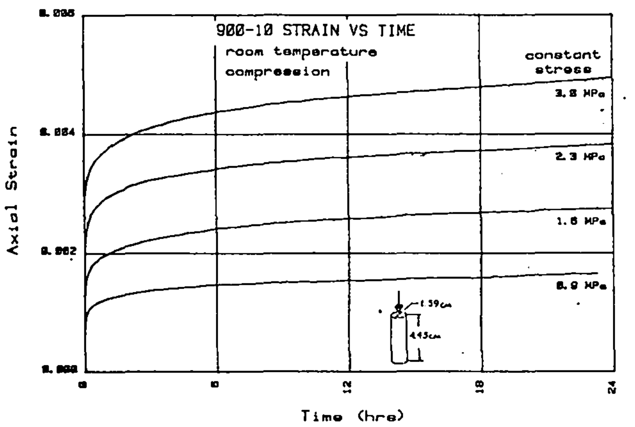

Fig. 1.
Total compressive strain vs time for sustained constant load.

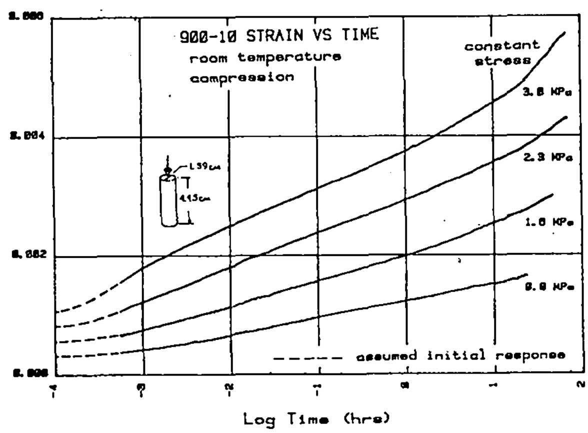

Fig. 2.
Total compressive strain vs log time for sustained constant load.

The first newly developed creep law was based upon a Prony series fit to the experimental creep data and has the form

$$\varepsilon_c = A + B_1\exp(-R_1 t) + B_2\exp(-R_2 t)$$

$$+ B_3\exp(-R_3 t) + B_4\exp(-R_4 t) + Ct, \quad (4)$$

where

$$A = -(B_1 + B_2 + B_3 + B_4), \quad (5a)$$

$$B_1 = A_1\sigma^{A_2}, \quad (5b)$$

$$B_2 = A_3\sigma^{A_4}, \quad (5c)$$

$$B_3 = A_5\exp(A_6\sigma), \quad (5d)$$

$$B_4 = A_7\sigma^{A_8}, \quad (5e)$$

and

$$C = A_9\exp(A_{10}\sigma), \quad (5f)$$

with the units of σ and t being pounds per square inch and hours, respectively. By choosing an appropriate functional form for each of the constants in Eq. (4), as demonstrated in Eq. (5), each constant in Eq. (5) was evaluated by using a least squares curve fit. The four creep tests provided four data points. The values of R_1 through R_4 were preset to 0.01, 0.1, 1.0 and 10.0, based upon earlier curve fitting analyses. The resulting values of the constants in Eq. (5) are:

$A_1 = -3.2517\times10^{-5}$ $\quad A_6 = 1.8759\times10^{-3}$

$A_2 = 5.8234\times10^{-1}$ $\quad A_7 = -5.8980\times10^{-6}$

$A_3 = 2.6708\times10^{-6}$ $\quad A_8 = 8.2636\times10^{-1}$

$A_4 = 9.0099\times10^{-1}$ $\quad A_9 = -1.0179\times10^{-5}$

$A_5 = 2.4385\times10^{-4}$ $\quad A_{10} = -5.4725\times10^{-3}$.

This model reproduces the constant load creep curves from which it was derived and predicts creep response for uniaxial creep specimens subjected to varying sustained load levels, but because of the nature of the formulation, it could not be employed in the Hewlett-Packard finite element model to reproduce the constant-strain rate-to-failure curves of Fig. 4. Attempts at doing so resulted in overflow errors associated with the nature of the loading. Therefore, we concluded that Eq. (4) provided a model of limited application in its present state.

We then developed a creep model of a somewhat different nature. Again, based upon the test results shown in Fig. 1 and 2, an expression of the form

$$\varepsilon_c = a_0\sigma^{a_1}[1-\exp(-a_2 t)] + a_3 t \quad (6)$$

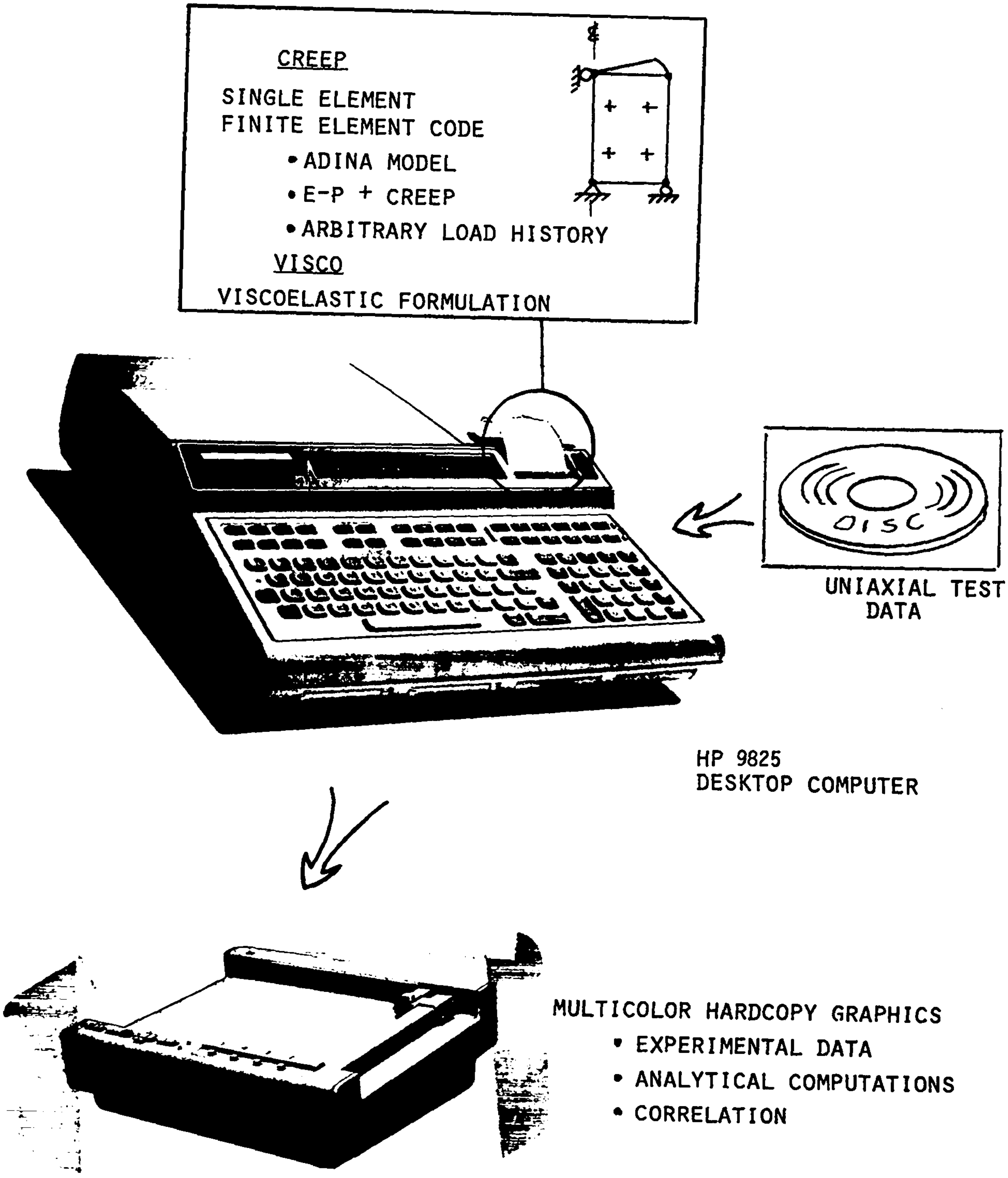

Fig. 3.
Hewlett-Packard system used to display test results and develop finite element models.

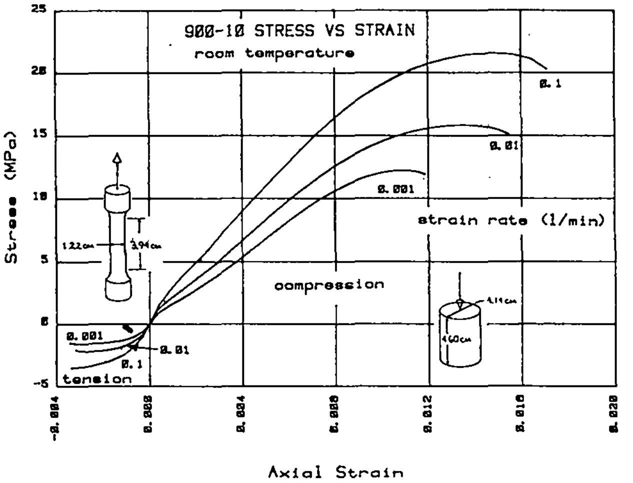

Fig. 4.
Constant-strain rate-to-failure experimental test results.

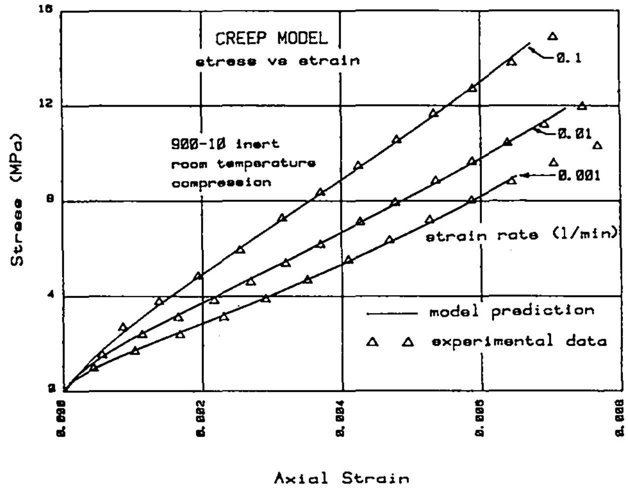

Fig. 5.
Constant-strain rate-to-failure predictions using Eq. (6).

was proposed, with a_0 through a_3 being material parameters, and with σ and t having units of pounds per square inch and hours, respectively. When we used this model to reproduce the stress-strain curves of Fig. 4 with the Hewlett-Packard finite element program, we found that values of 5.658 x 10^{-5}, 0.7, and 7.5 x 10^{-8} for a_0, a_1, and a_3, respectively, are satisfactory, but the parameter a_2 is rate sensitive. This parameter has the form

$$a_2 = \alpha - \beta \sigma. \quad (7)$$

The rate dependence of the α and β terms is shown in Table I. These values were used to obtain the stress-strain curves shown in Fig. 5. In the Hewlett-Packard finite element program, the values of the initial modulus of elasticity E_0 and Poisson's ratio γ were 600,000 psi and 0.36, respectively. Both values were obtained from prior testing and analysis. The

TABLE I

RATE-DEPENDENT PARAMETRIC VALUES

	Strain Rate (min^{-1})		
	0.001	0.01	0.1
α	14	80	400
β	0.008	0.032	0.132

analysis also was based upon an assumption of incompressible creep strains, which is consistent with the ADINA theory that it duplicates. For simplicity, these analyses were limited to elastic, rather than elastic-plastic, material behavior.

The excellent agreement between corresponding curves in Figs. 4 and 5 supports the formulation of Eq. (6) as being representative of material behavior over a restricted range. The limitation of the programmed material behavior to the elastic range was stated earlier. This model was also extrapolated to the case of constant sustained loading (~zero strain rate) with results that appear to correlate with measured creep data; however, this correlation has not been completely checked at this time. We strongly recommended that this model be further studied and developed. The Hewlett-Packard finite element program is undergoing an extension from the initial elastic range limitation to the general elastic-plastic modeling capability. It provides the developer with an extremely efficient means of exercising in-depth any proposed creep model, such as the one of Eq. (6), relative to time-step sensitivity, load function, etc.

B. Viscoelastic Model Formulation

Additional studies were focused on the development of viscoelastic material behavior models. This was partly based upon shortcomings associated with equation-of-state creep theory models, e.g., lack of a capability of recovery behavior prediction. Because the degree of viscoelastic linearity of the HE was an unknown, the general overall approach was considered to be nonlinear. Nonlinear viscoelastic theory is highly formulated,[3] but applied with difficulty. We felt that a sensible approach to the development of a viscoelastic model is to initially formulate a linear model that, as will be seen, provides the basis for a nonlinear analysis. After determining how well this model portrays the material response, the direction of further development will become more clear.

The nonlinear viscoelastic constitutive theory developed by Schapery[4] was derived by the use of principles of the thermodynamics of irresversible processes. In this formulation, the nonlinearity is contained in a "reduced time," where the reduced time is an implicit function of stress. It is based, in part, on the observation that nonlinear stress relaxation of various materials can be described in terms of the same time-dependent properties found in the linear range. The corresponding creep expression is:

$$\varepsilon(t) = g_0\, J_0\, \sigma(t) + g_1 \int_0^t \phi(\xi_\sigma - \xi^1_\sigma) \frac{\partial g_2 \sigma(\tau)}{\partial \tau}\, d\tau . \qquad (8)$$

This form is employed by Schapery under conditions of constant temperature and uniaxial stress σ, where

$$\xi_\sigma = \xi_\sigma\,(t) = \int_0^t \frac{ds}{a_\sigma[\sigma(s)]} \qquad (9a)$$

and

$$\xi^1_\sigma = \xi_\sigma\,(\tau) = \int_0^\tau \frac{ds}{a_\sigma[\sigma(s)]} \qquad (9b)$$

are the reduced times and a_σ is a shift factor. The terms g_0, g_1, g_2, and a_σ are functions of stress at constant temperature; J_0 and ϕ (t) are the time-independent compliance and the time-dependent creep function, respectively, and τ and s are generic times.

None of the parameters in Eq. (8) are fundamental physical or thermodynamic constants and, therefore, all must be determined from creep, relaxation, or other mechanical tests of the material being considered.

For linear viscoelastic behavior, with $g_0 = g_1 = g_2 = a_\sigma = 1$, Eq. (8) reduces to

$$\varepsilon(t) = J_0\, \sigma(t) + \int_0^t \phi\, (t-\tau)\, \frac{\partial \sigma(\tau)}{\partial \tau}\, d\tau , \qquad (10)$$

or the equivalent form

$$\varepsilon(t) = \int_0^t J(t-\tau)\, \frac{\partial \sigma(\tau)}{\partial \tau}\, d\tau . \qquad (11)$$

In developing an expression for J(t), the creep compliance, the creep test data shown in Figs. 1 and 2 were combined, and produced the somewhat surprising narrow data band shown in Fig. 6. By using a Prony series curve fit, the following expression was obtained.

$$J(t) = \rho + \rho_1 \exp(-\delta_1 t) + \rho_2 \exp(-\delta_2 t) + \rho_3 \exp(-\delta_3 t) + \rho_4 \exp(-\delta_4 t) + \rho_5 \exp(-\delta_5 t) + \rho_6 t , \qquad (12)$$

with the units of t being hours, and where

ρ = .0000211956

ρ_1 = -.0000111843

ρ_2 = -.0000004538

ρ_3 = -.0000023488

ρ_4 = -.0000009812

ρ_5 = -.0000035347

ρ_6 = -.0000000246,

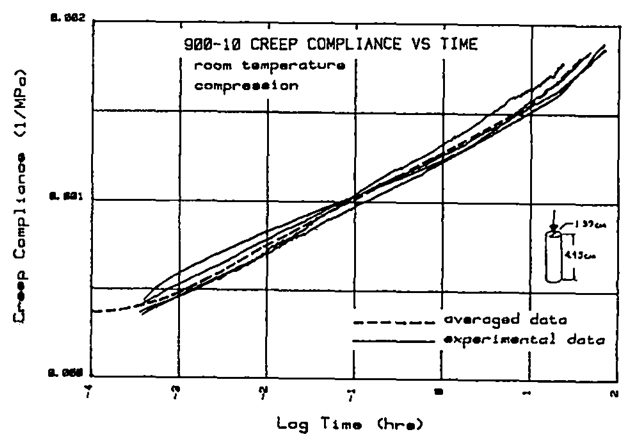

Fig. 6.
Creep compliance vs log time for four tests.

with the values of δ_1 through δ_5 preset to 0.01, 0.1, 1.0, 10.0 and 100.0.

A linear viscoelastic program, using the expression for J(t) from Eq. (12) and the loading from Fig. 7, was written for the Hewlett-Packard calculator discussed earlier, and used to predict the experimental stress-strain behavior shown in Fig. 8. The results of the program are shown in Fig. 9. The similarity between the two curves (Figs. 8 and 9) is interesting. Notably, the slopes of the two curves are approximately the same, after two pulses. However, the viscoelastic model predicts complete strain recovery between pulses, whereas the actual test data show this not to be the case. Comparison of the results obtained with the linear viscoelastic model to the experimental data might lead one to postulate a new "damage theory." However, refinement of this model might be expected to yield a damage-like response.

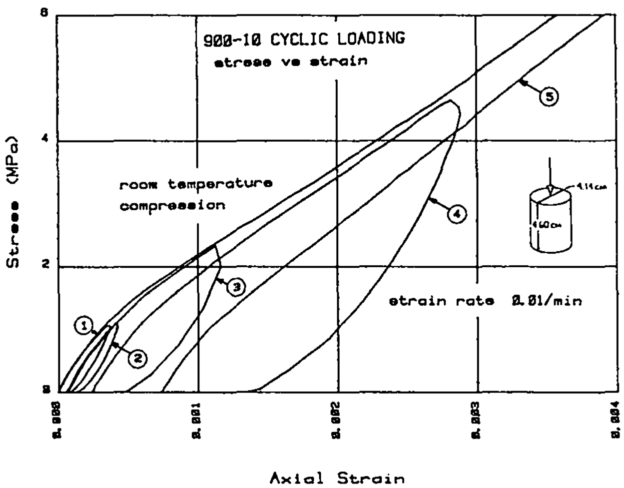

Fig. 8.
Stress-strain diagram corresponding to experimental pulsed pressure loading of Fig. 7.

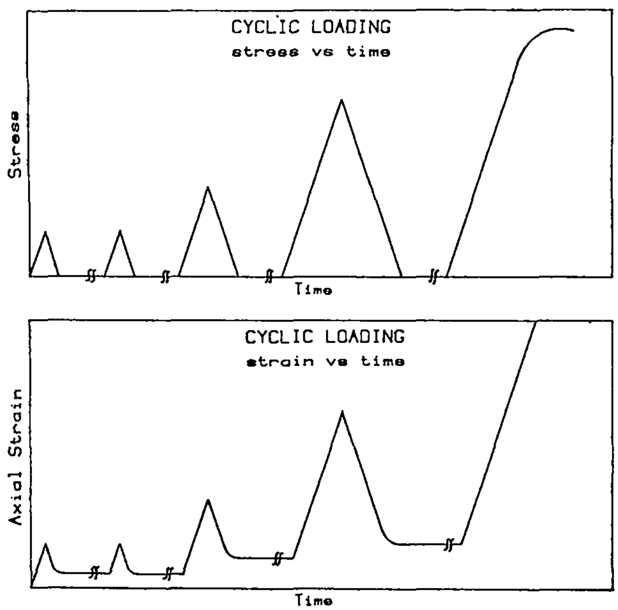

Fig. 7.
Pulsed pressure loading with extended recovery time between pulses.

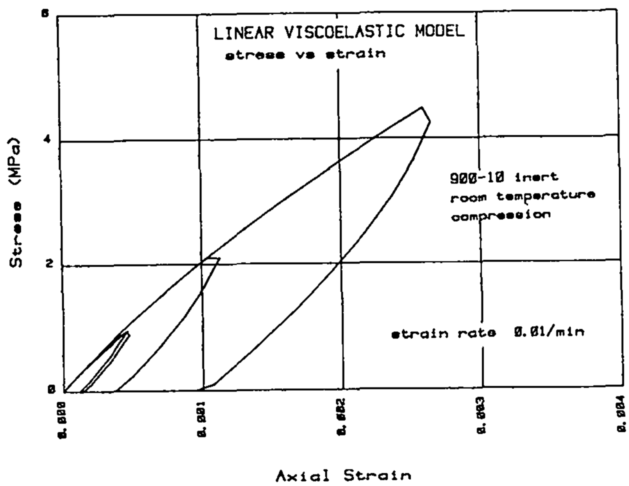

Fig. 9.
Stress-strain diagram predicted using Eq. (11).

III. SUMMARY AND CONCLUSIONS

Three time-dependent constitutive behavior models have been formulated. Clearly, all are limited and exploratory. Equation (4) predicts the time-dependent behavior under sustained constant load. Equation (6) correlates extremely well with test data based on constant-strain rate-to-failure testing in the "elastic range," but is, in its present form, strain-rate dependent. Equation (6) also appears to accurately predict time-dependent behavior under conditions of constant load. Equation (11), a linear viscoelastic model, provides a reasonable comparison with experimental data on cyclic loading and poses some very interesting questions related to damage.

These simple analytical models, together with the Hewlett-Packard finite element program, can be utilized to gain more valuable insight into the constitutive behavior of this class of materials.

ACKNOWLEDGMENT

The authors gratefully acknowledge the efforts of Harry Duhamel who helped conduct the experimental studies.

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